Basics of Vehicle Dynamics

Lateral dynamics: steady-state cornering
Introduction: steering with and w/o tyre side slip

Instantaneous centre changes place due to tyre side slip.
**Introduction: kinematic (“Ackermann”) steering**

Cornering at low speed, negligible side forces (and therefore side slip)

Steering mechanism configuration named after Rudolph Ackermann (patent agent), invented by Georg Lankensperger XIX century (Germany)
Introduction: high speed cornering

Tyre side forces introduce side slip
Modifying motion direction

Migration of instantaneous centre changes curvature radius and therefore cornering “sharpness”
Introduction: “bicycle model”

“Effective wheel” front

“Effective wheel” rear

CM

CM
Cornering at low speed

\[ \Delta \text{ OCA: } \frac{\sin(\theta_A - \beta)}{a} = \frac{\sin(\pi/2 - \theta_A)}{R_c} \]

\[ \Delta \text{ OCB: } \sin \beta = \frac{b}{R_K} \]

\[ \frac{\sin \theta_A \cos \beta - \cos \theta_A \sin \beta}{a} = \frac{\cos \theta_A}{R_c} \]

\[ \tan \theta_A \cos \beta - \sin \beta = \frac{a}{R_c} \]

\[ \sin \beta = \frac{b}{R_c} \]

\[ \tan \theta_A \cos \beta = \frac{l}{R_c} \]
Cornering at low speed

\[ \tan \theta_A \cos \beta = \frac{l}{R_C} \]

Further we assume:

- \( R_C \gg l \)
- angles \( \theta_A, \beta \) are small \( \Rightarrow \tan \theta_A \approx \theta_A \text{ (radians!)}, \cos \beta \approx 1 \)

\[ \theta_A = \frac{l}{R_C} \]

\( \rightarrow \) Wheelbase of the vehicle
\( \rightarrow \) CM’s trajectory curvature radius
\( \rightarrow \) Required steering angle
Steady-state cornering

Cornering at high speed

\[
\frac{\sin((\theta - \delta_r) - \beta)}{\sin(\pi/2 - (\theta - \delta_f))} = \frac{a}{R_c}
\]

\[
\frac{\sin(\delta_r + \beta)}{\sin(\pi/2 - \delta_f)} = \frac{b}{R_c}
\]

\[
\sin (\pi/2 - x) = \cos x
\]

Small angles: \( \sin x \approx x, \cos x \approx 1 \)

\[
\theta = \frac{l}{R_c} + \delta_f - \delta_r
\]
Cornering at high speed

- Three types of steering behavior

\[ \theta = \frac{l}{R_C} + \delta_f - \delta_r \]

\[ \theta_A = \frac{l}{R_C} \]

To achieve the same \( R_C \), driver has to turn the steering wheel:

- \( \delta_f > \delta_r \) UNDERSTEER MORE
- \( \delta_f = \delta_r \) NEUTRAL STEER EQUAL
- \( \delta_f < \delta_r \) OVERSTEER LESS
Cornering at high speed

- Three types of steering behavior

![Diagram of steering behaviors](autozine.org)

![Diagram of vehicle dynamics](pussiriot.wordpress.com)
Basic analysis of steering behavior

- Assumption: small angles, large curve radius
- Constant velocity \( v \), constant \( \theta \) \( \Rightarrow \) constant \( R_C \)

\[ F_C = m \frac{v^2}{R_C} \]  
"centrifugal force“  
(D'Alembert's principle of inertial forces)
Basic analysis of steering behavior

\[ F_{yp} + F_{yz} = m \frac{v^2}{R_c} \]
\[ a \cdot F_{yp} = b \cdot F_{yz} \]
\[ a + b = l \quad - \text{wheelbase} \]

Further we apply:

\[ W_f = \frac{b}{l} \cdot W \]
\[ \frac{b}{l} = \frac{W_f}{W} \]
\[ W_f = \frac{a}{l} \cdot W \]
\[ \frac{a}{l} = \frac{W_r}{W} \]

Finaly:

\[ F_{yf} = \frac{W_f}{W} \frac{m \cdot v^2}{R_c} \]
\[ F_{yr} = \frac{W_r}{W} \frac{m \cdot v^2}{R_c} \]

Important:

\[ \frac{F_{yf}}{F_c} = \frac{W_f}{W} \]
\[ \frac{F_{yr}}{F_c} = \frac{W_r}{W} \]
\[ \frac{F_{yr}}{F_{yf}} = \frac{W_r}{W} \]
Basic analysis of steering behavior

For moderate $F_y$ (small $\delta$): $F_y = c_\delta \cdot \delta$

$$F_{yf} = \frac{W_f m \cdot v^2}{W \cdot R_C}$$  
$$F_{yr} = \frac{W_r m \cdot v^2}{W \cdot R_C}$$

Linear approximation:

$$F_y = c_\delta \cdot \delta$$

Required steering angle

$$\delta_f = \frac{W_f \cdot \frac{v^2}{C_{\delta f} \cdot R_C \cdot g}}{1}$$

$$\delta_r = \frac{W_r \cdot \frac{v^2}{C_{\delta r} \cdot R_C \cdot g}}{1}$$

$$\theta = \frac{l}{R_C} + \delta_f - \delta_r$$

$$\theta = \frac{l}{R_C} + \frac{W_f \cdot v^2}{C_{\delta f} \cdot R_C \cdot g} - \frac{W_r \cdot v^2}{C_{\delta r} \cdot R_C \cdot g}$$
Basic analysis of steering behavior

\[ \theta = \frac{l}{R_C} + \left( \frac{W_f}{C_{\delta f}} - \frac{W_r}{C_{\delta r}} \right) \cdot \frac{v^2}{g \cdot R_C} \]

\[ \theta = \frac{l}{R_C} + K \cdot \frac{a_y}{g} \]

\[ K = \frac{W_f}{C_{\delta f}} - \frac{W_r}{C_{\delta r}} \]

Solution for most simple model – comprises most fundamental factors

Further factors affecting steering behavior:

- Wheel alignment and elastokinematics of the wheel suspension
- Distribution of vertical loads amongst wheels at the same axle
- Non-linear tyre behavior
- Presence of driving/braking torque (i.e. longitudinal tyre force)
Basic analysis of steering behavior

\[ \theta = \frac{l}{R_c} + K \cdot \frac{a_y}{g} \]

- \( K > 0 \) - UNDERSTEER
- \( K = 0 \) - NEUTRAL
- \( K < 0 \) - OVERSTEER

Oversteer vehicle is unstable by nature above critical speed!

Drift video
Basic analysis of steering behavior

• Cornering at high speed: impact of load distribution – “steering by gas-pedal”

\[ K = \frac{W_f}{C_{\delta f}} - \frac{W_r}{C_{\delta r}} \]

→ Braking/accelerating: redistribution of \( W_f / W_r \)

→ Tyre load \( (W_T) \) influences \( c_{\delta} \) (larger \( W_T \) ⇒ larger \( c_{\delta} \))

→ Influence is degressive!

→ Deccelerating (braking) in the curve ⇒ car behavior changes towards more oversteer

→ Accelerating in the curve ⇒ car behavior changes towards more understeer

☞ “Lift-off oversteer” video
Basics of Vehicle Dynamics

Lateral dynamics: transient maneuvers
Equations of motion

- General equations of planar motion

\[ m \cdot \ddot{a}_C = \Sigma \vec{F}_i \]  \hspace{1cm} (1,2)

\[ J_{Cz} \cdot \ddot{\phi} = \Sigma \vec{M}_C \]  \hspace{1cm} (3)

\[ \ddot{a}_C = \ddot{a}_{CT} + \ddot{a}_{CN} = \ddot{a}_{Cx} + \ddot{a}_{Cy} \rightarrow \text{Vector equation to scalar (axes: T/N or x/y)} \]

\[ a_{CN} = \frac{v^2}{\rho} \hspace{1cm} a_{CT} = \frac{dv}{dt} = \dot{v} \]

D'Alembert's principle:

\[ \vec{F}_{IN} = -m \cdot \ddot{a}_C = -m \cdot \ddot{a}_T + \left( -m \cdot \ddot{a}_N \right) \]

\[ \Sigma \vec{F}_i + \Sigma \vec{F}_{IN} = 0 \Rightarrow -m \cdot \ddot{a}_{CT} - m \cdot \ddot{a}_{CN} + \Sigma \vec{F}_i = 0 \]
Equations of motion

• Kinematic parameters and forces

\[
v = R_C \cdot \dot{\phi} = \rho \cdot (\dot{\phi} + \dot{\beta})
\]

\[
\begin{align*}
v_f^C &= l_f \cdot \dot{\phi} \\
v_r^C &= l_r \cdot \dot{\phi}
\end{align*}
\]
Transient maneuvers

**Equations of motion**

\[
x: \quad -m \cdot \dot{v} \cdot \cos \beta + m \cdot \frac{v^2}{\rho} \cdot \sin \beta + F_{xf} \cdot \cos \theta - F_{yf} \cdot \sin \theta + F_r - F_w = 0
\]

\[
y: \quad m \cdot \dot{v} \cdot \sin \beta + m \cdot \frac{v^2}{\rho} \cdot \cos \beta - F_{xf} \cdot \sin \theta - F_{yf} \cdot \cos \theta - F_{yr} \pm F_{Ay} = 0
\]

\[
(3) \quad -J_{cz} \cdot \dot{\phi} + l_f \cdot (F_{xf} \cdot \sin \theta + F_{yf} \cdot \cos \theta) - l_r \cdot F_{yr} \pm A \cdot F_{Ay} = 0
\]

\[
F_y = F_y(\delta)
\]

\[
\theta = \theta(t)
\]

\[
F_x, v(t) - assumptions...
\]
Transient maneuvers

Equations of motion

- Special case for constant velocity:
  - \( \frac{d\beta}{dt} \) and \( \frac{d\phi}{dt} \) are directly related
- One ODE remains in the form of:
  \[
  A \cdot \ddot{\beta} + B \cdot \dot{\beta} + C \cdot \beta = k_1 \cdot \dot{\theta} + k_2 \cdot \theta
  \]
  With homogeneous part:
  \[
  A \cdot \ddot{\beta} + B \cdot \dot{\beta} + C \cdot \beta = 0
  \]
  \[\Rightarrow \text{Vehicle can undergo damped angular oscillations}\]
Some typical transient maneuvers

- Harmonic input
- Steering ramp
- Steering pulse
- Braking in the curve
- “Scandinavian flick”
- Sudden side wind gust
- etc.
Transient maneuvers

Steering ramp

Steering ramp @ 1 sec
Transient maneuvers

Braking in the curve

15 sec: Steering ramp

30 sec: Brake on

10 sec: Throttle off
Braking in the curve – impact of wheel lock

Front wheel lock ⇒ UNSTEERABLE, STABLE VEHICLE

Rear wheel lock ⇒ STEERABLE, UNSTABLE VEHICLE!

couple

Lateral forces at both axles – STEERABLE AND STABLE VEHICLE

More favorable situation for untrained driver!
(Higher probability of avoiding the accident)

\[
\frac{m \cdot v^2}{R_C}
\]
Transient maneuvers

Sudden side wind with steering

Sudden wind gust @ 2 sec
“Scandinavian flick”

“Scandinavian flick” video
Basics of Vehicle Dynamics
Vehicle vibrations and vertical dynamics
Basic modelling approaches
Topics of interest

Equations for quarter-car model

\[ \omega_n = \sqrt{\frac{RR}{M}} \]  
(radians/sec)  
Natural frequency

\[ \omega_d = \omega_n \sqrt{1 - \zeta_s^2} \]  
Damped frequency

\[ \zeta_s = \text{Damping ratio} \]  
\[ = \frac{C_s}{\sqrt{4 K_s M}} \]  
\[ C_s = \text{Suspension damping coefficient} \]

Equations for quarter-car model

\[ M \ddot{Z} + C_s \dot{Z} + K_s Z = C_s \dot{Z}_u + K_s Z_u + F_b \]

\[ m \ddot{Z}_u + C_s \dot{Z}_u + (K_s + K_t) Z_u = C_s \dot{Z} + K_s Z + K_t Z_r + F_w \]

Vehicle vibrations

Main I/O relations

Vehicle vibrations

Main I/O relations

Body natural frequency: \( \sim 1.5 \text{ Hz} \)
Wheel natural frequency: \( \sim 10\div15 \text{ Hz} \)

Impact of stiffness

Vehicle vibrations

Impact of damping

Impact of suspension and tyre elasticity on dynamic axle load gain
Basics of Vehicle Dynamics

Further reading
Further reading

If this was interesting for you...

- ...and plenty more!
THE END
THANKS FOR YOUR ATTENTION

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